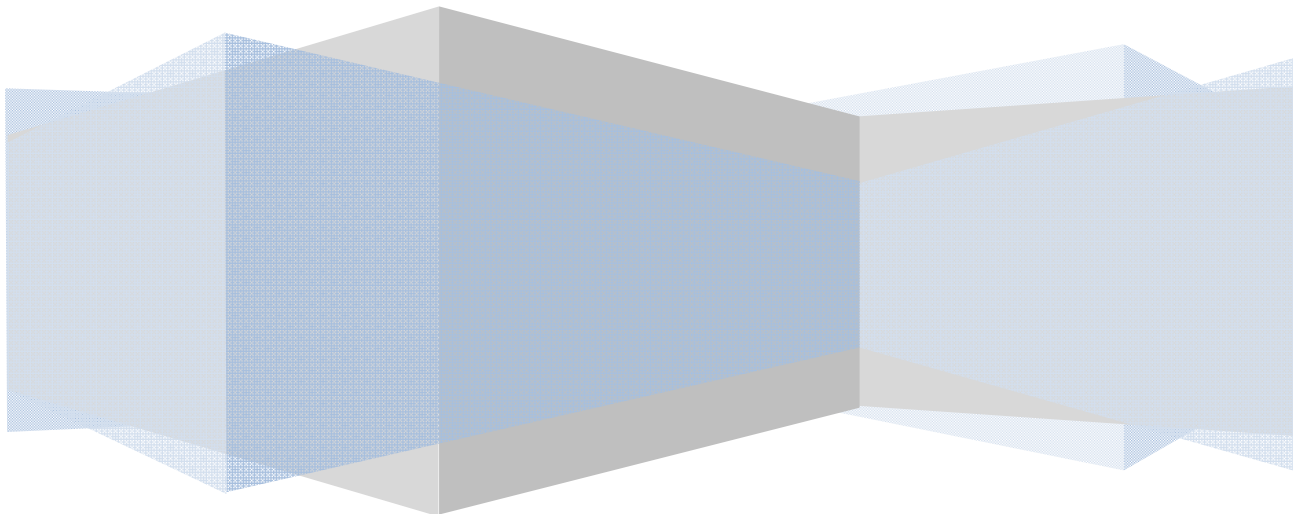


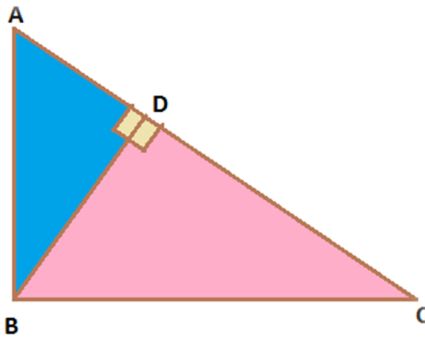
Class Notes on Pythagoras Theorem (Chapter 12)

Chapter - 12 English Version



Pythagoras Theorem

In a right angled triangle, the square on the hypotenuse is equal to the sum of the square on the other two sides.



Data: In $\triangle ABC$, $\angle ABC = 90^\circ$

To prove : $AB^2 + BC^2 = CA^2$

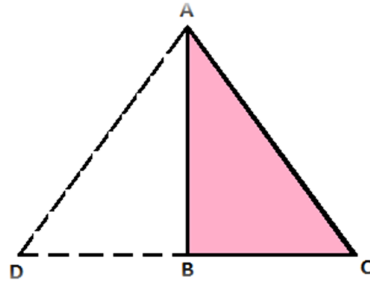
Construction : Draw $BD \perp AC$.

Proof

In $\triangle ABC$ and $\triangle ADB$	
$\angle ABC = \angle ADB = 90^\circ$	Data and construction
$\angle BAD = \angle BAD$	Common angle
$\therefore \triangle ABC \sim \triangle ADB$	Equiangular triangles
$\Rightarrow \frac{AB}{AD} = \frac{AC}{AB}$	
$\Rightarrow AB^2 = AC \cdot AD \dots (1)$	
$\triangle ABC$ ಮತ್ತು $\triangle BDC$ ಗಳಲ್ಲಿ	
$\angle ABC = \angle BDC = 90^\circ$	Data and construction
$\angle ACB = \angle ACB$	Common angle
$\therefore \triangle ABC \sim \triangle BDC$	Equiangular triangles
$\Rightarrow \frac{BC}{DC} = \frac{AC}{BC}$	
$\Rightarrow BC^2 = AC \cdot DC \dots (2)$	
$AB^2 + BC^2 = (AC \cdot AD) + (AC \cdot DC) [\because (1) + (2)]$	
$AB^2 + BC^2 = AC \cdot (AD + DC)$	
$AB^2 + BC^2 = AC \cdot AC$	$AD + DC = AC$
$AB^2 + BC^2 = AC^2$	

**Converse of
Pythagoras Theorem**

"If the square on the longest side of a triangle is equal to the sum of the squares on the other two sides, then those two sides contain a right angle."



Data: In $\triangle ABC$, $AB^2 + BC^2 = AC^2$

To Prove : $\angle ABC = 90^\circ$

Construction: Draw perpendicular to AB at B. Select a point D on it such that, $DB = BC$. Join 'A' and 'D'

Proof:

In $\triangle ABD$	
$\angle ABC = 90^\circ$	Construction
$\therefore AD^2 = AB^2 + BC^2$	Pythagoras theorem
But, in $\triangle ABC$	
$AC^2 = AB^2 + BC^2$	Given
$\Rightarrow AD^2 = AC^2$	
$\therefore AD = AC$	
In $\triangle ABD$ and $\triangle ABC$,	
$AD = AC$	Proved
$BD = BC$	Construction
$AB = AB$	Common sides
$\triangle ABD \cong \triangle ABC$	S.S.S.
$\Rightarrow \angle ABD = \angle ABC$	
ಆದ್ದರಿಂದ, $\angle ABD + \angle ABC = 180^\circ$	Complimentary angles
$\Rightarrow \angle ABD = \angle ABC = 90^\circ$	

Pythagorean triplets

3 4 5

7 24 25

6 8 10

8 15 17

9 12 15

10 24 26

15 20 25

5 12 13

12 16 20

18 24 30

ILLUSTRATIVE EXAMPLES

Numerical problems based on Pythagoras theorem

Example 1: In a right angled ΔABC , $\angle B = 90^\circ$, $AC = 17\text{cm}$ and $AB = 8\text{cm}$, find BC

Sol: Given, in ΔABC , $\angle B = 90^\circ$

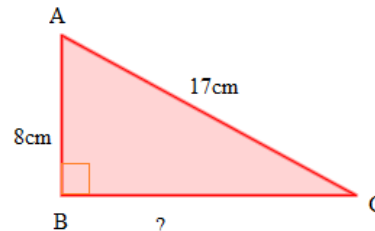
$\therefore AC^2 = AB^2 + BC^2$ [\because Pythagoras theorem]

$\Rightarrow BC^2 = AC^2 - AB^2$

$BC^2 = 17^2 - 8^2$

$BC^2 = 289 - 64 = 225$

$\therefore BC = 225 \text{ cm}$



Example 2: In ΔABC , $\angle ABC = 45^\circ$, $AM \perp BC$, $AM = 4\text{cm}$ and $BC = 7\text{cm}$. Find the length of AC

In $\triangle AMB$, $\angle AMB = 90^\circ$, $\angle ABM = 45^\circ \therefore \angle BAM = 90^\circ$

$\triangle AMB$ is an isosceles right angled triangle

$\therefore MA = MB = 4\text{cm}$

$MC = BC - MB = 7 - 4 = 3\text{cm}$

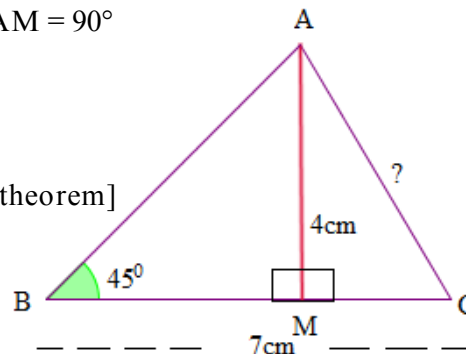
$\therefore MC = 3\text{cm}$

In $\triangle AMC$, $AC^2 = AM^2 + MC^2$ [\because Pythagoras theorem]

$AC^2 = 4^2 + 3^2$

$AC^2 = 16 + 9$

$AC^2 = 25$



AC = 5cm

Example3: In the rectangle WXYZ, XY + YZ = 17cm and XZ + YW = 26cm. calculate the length and breadth of the rectangle.

Sol: XZ + YW = 26cm

$$d_1 + d_2 = 26\text{cm}$$

$$2d = 26\text{cm}$$

$$d = 13\text{cm}$$

$$\therefore XZ = YW = 13\text{cm}$$

Let length = XY = x cm \Rightarrow breadth = XW = (17-x)cm

In $\triangle WXY$,

$$WX^2 + XY^2 + WY^2 \quad [\because \text{Pythagoras theorem}]$$

$$(17-x)^2 + x^2 = 13^2$$

$$(289 - 34x + x^2) + x^2 = 169$$

$$(2x^2 - 34x + 120 = 0) \div 2$$

$$x^2 - 17x + 60 = 0$$

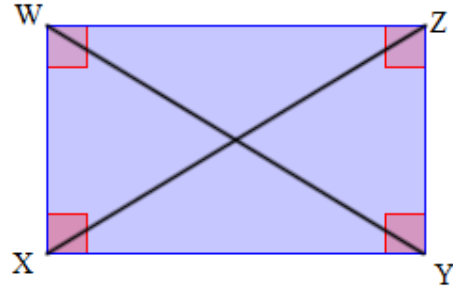
$$x^2 - 12x - 5x + 60 = 0$$

$$x(x-12) - 5(x-12) = 0 \Rightarrow (x-12)(x-5) = 0$$

$$\Rightarrow x - 12 = 0 \text{ or } x - 5 = 0$$

$$\Rightarrow x = 12 \text{ or } x = 5$$

Length = 12cm, breadth = 5cm



Example4: An insect 8 m away from the foot of a lamp post which is 6m tall, crawls towards it. After moving through a distance, its distance from the top of the lamp post is equal to the distance it has moved. How far is the insect away from the foot of the lamp post? [Bhaskaracharya's Leelavathi]

Sol: Distance between the insect and the foot of the lamp post = BD = 8m.

The height of the lamp post = AB = 6m.

After moving a distance, let the insect be at C,

Let AC = CD = x m

$$\therefore BC = (8 - x) \text{ m.}$$

In $\triangle ABC$, $\angle B = 90^\circ$

$$\therefore AC^2 = AB^2 + BC^2 \quad [\because \text{Pythagoras theorem}]$$

$$x^2 = 6^2 + (8 - x)^2$$

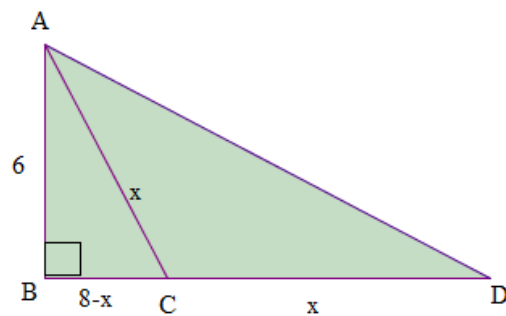
$$x^2 = 36 + 64 - 16x + x^2$$

$$\therefore 16x = 100$$

$$x = 6.25\text{m}$$

$$\therefore BC = 8 - x = 8 - 6.25 = 1.75\text{m}$$

The insect is 1.75m away from the foot of the lamp post



Riders based on Pythagoras Theorem

Example5: In the given figure, $AD \perp BC$, Prove that $AB^2 + CD^2 = BD^2 + AC^2$

In $\triangle ADC$, $\angle ADC = 90^\circ$

$$\therefore AC^2 = AD^2 + CD^2 \quad [\because \text{Pythagoras theorem}] \text{-----(1)}$$

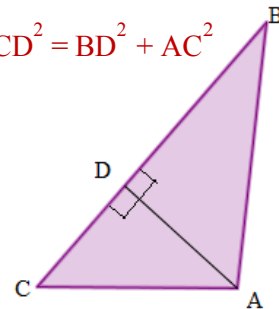
In $\triangle ADB$, $\angle ADB = 90^\circ$

$$\therefore AB^2 = AD^2 + BD^2 \quad [\because \text{Pythagoras theorem}] \text{-----(2)}$$

Subtracting (1) from (2), we get

$$AB^2 - AC^2 = BD^2 - CD^2$$

$$AB^2 + CD^2 = BD^2 + AC^2$$



Example6: In $\triangle ABD$, C is the point on BD such that $BC:CD = 1:2$ and $\triangle ABC$ is an equilateral triangle. Prove that $AD^2 = 7AC^2$

Sol: Data: In $\triangle ABD$, $BC:CD = 1:2$

In $\triangle ABC$, $AB = BC = CA$

To prove: $AD^2 = 7AC^2$

Construction: Draw $AE \perp BC$

Proof: In $\triangle ABC$,

$$BE = EC = \frac{a}{2} \text{ and } AE = \frac{a\sqrt{3}}{2}$$

In $\triangle AED$, $\angle AED = 90^\circ$ [\because construction]

$$AD^2 = AE^2 + ED^2 \quad [\because \text{Pythagoras theorem}]$$

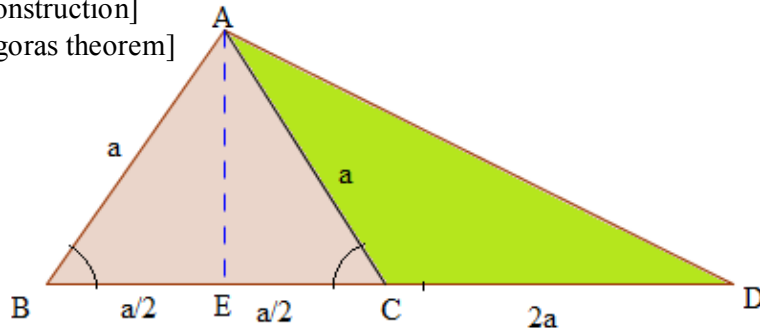
$$AD^2 = \left(\frac{a\sqrt{3}}{2}\right)^2 + \left(2a + \frac{a}{2}\right)^2$$

$$AD^2 = \frac{3a^2}{4} + \left(\frac{5a}{2}\right)^2$$

$$AD^2 = \frac{3a^2}{4} + \frac{25a^2}{4}$$

$$AD^2 = \frac{28a^2}{4}$$

$$AD^2 = 7AC^2$$



Exercise 12.1

Numerical problems based on Pythagoras theorem

1. The sides of a right angled triangle containing the right angle are 5cm and 12cm, find its hypotenuse .

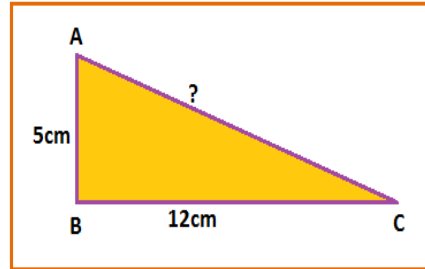
$$AC^2 = AB^2 + BC^2$$

$$AC^2 = 5^2 + 12^2$$

$$AC^2 = 25 + 144$$

$$AC^2 = 169$$

$$AC = 13\text{cm}$$



2. Find the length of the diagonal of a square of side 12cm.

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = 12^2 + 12^2$$

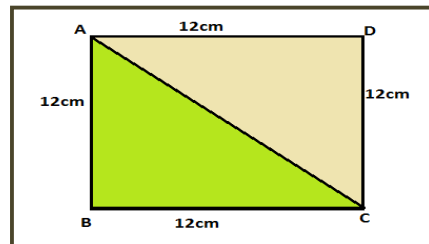
$$AC^2 = 144 + 144$$

$$AC^2 = 288$$

$$AC = \sqrt{288}$$

$$AC = \sqrt{2 \times 144}$$

$$AC = 12\sqrt{2} \text{ cm}$$



3. The length of the diagonal of a rectangular playground is 125m and the length of one side is 75m. Find the length of the other side .

$$AC^2 = AB^2 + BC^2$$

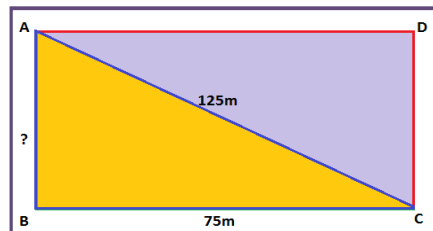
$$125^2 = AB^2 + 75^2$$

$$15625 = AB^2 + 5625$$

$$AB^2 = 15625 - 5625$$

$$AB^2 = 10000$$

$$AB = 100$$



4. In $\triangle LAW$, $\angle LAW = 90^\circ$, $\angle LNA = 90^\circ$ $LW = 26\text{cm}$, $LN = 6\text{cm}$ ಮತ್ತು $AN = 8\text{cm}$, Calculate the length of WA.

ΔLNA ಯಲ್ಲಿ, $\angle LNA = 90^\circ$

$$\therefore LA^2 = LN^2 + NA^2$$

$$\therefore LA^2 = 6^2 + 8^2$$

$$\therefore LA^2 = 36 + 64 = 100$$

$$\therefore LA = 10\text{cm}$$

ΔLAW ನಲ್ಲಿ, $\angle LAW = 90^\circ$

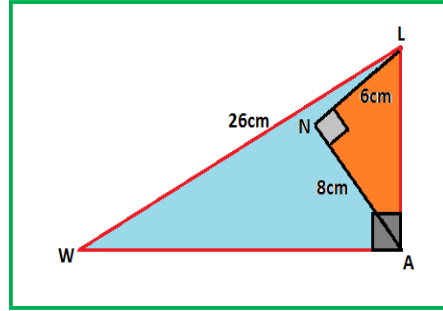
$$\therefore WA^2 = LW^2 + LA^2$$

$$\therefore WA^2 = 26^2 - 10^2$$

$$\therefore WA^2 = 676 - 100$$

$$\therefore WA^2 = 576$$

$$\therefore WA = 24\text{cm}$$



5. A door of width 6 meter has an arch above it having a height of 2 meter. Find the radius of the arch .

In the figure, $OC = OB =$ ತ್ರಿಜ್ಯ = r

$$OC = r - 2$$

In ΔOMB , $\angle OMB = 90^\circ$

$$\therefore \text{Radius } OB^2 = OM^2 + MB^2$$

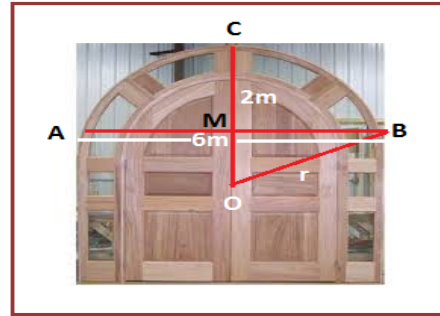
$$\therefore r^2 = (r - 2)^2 + 3^2$$

$$\therefore r^2 = r^2 - 4r + 4 + 9$$

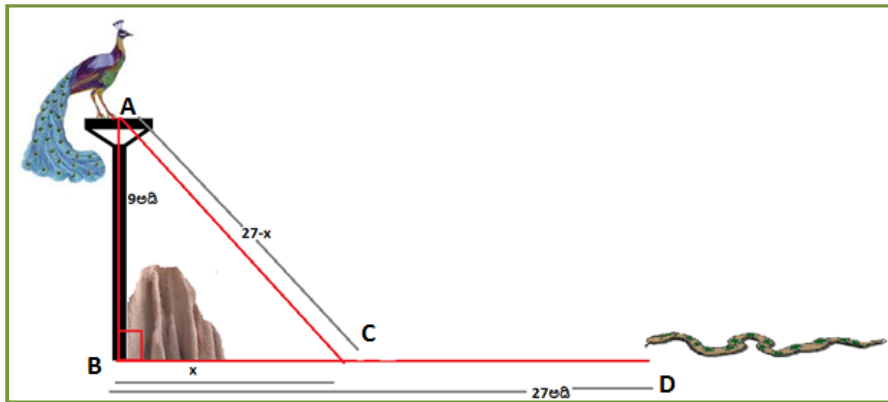
$$\therefore 4r = 4 + 9$$

$$\therefore r = \frac{13}{4}$$

$$\therefore r = 3.25\text{m}$$



6. A peacock on a pillar of 9 feet height on seeing a snake coming towards its hole situated just below the pillar from a distance of 27 feet away from the pillar will fly to catch it. If both possess the same speed, how far from the pillar they are going to meet?



In the figure, Pillar $AB = 9$ feet, $BD = 27$ feet, The distance travelled by (Peacock) snake DC (AC) = $27 - x$

$$BC = x \text{ feet}$$

In ΔABC , $\angle ABC = 90^\circ$

$$\therefore AC^2 = AB^2 + BC^2$$

$$\therefore (27 - x)^2 = 9^2 + x^2$$

$$\therefore 729 - 54x + x^2 = 81 + x^2$$

$$\therefore 729 - 54x = 81$$

$$\therefore 729 - 81 = 54x$$

$$\therefore 648 = 54x$$

$$\therefore x = \frac{648}{54}$$

$$\therefore x = 12 \text{ feet}$$

\therefore Peacock and snake meet 12 feet away from the pillar.

Riders based on Pythagoras theorem

1. $\triangle MGN$ ನಲ್ಲಿ $MP \perp GN$. If $MG = 'a'$ units, $MN = 'b'$ units, $GP = 'c'$ units, $PN = 'd'$ units But,

Prove that $(a + b)(a - b) = (c + d)(c - d)$.

$\triangle MPG$ ಯಲ್ಲಿ, $\angle MPG = 90^\circ$,

$$\therefore MP^2 = a^2 - c^2 \text{-----(1)}$$

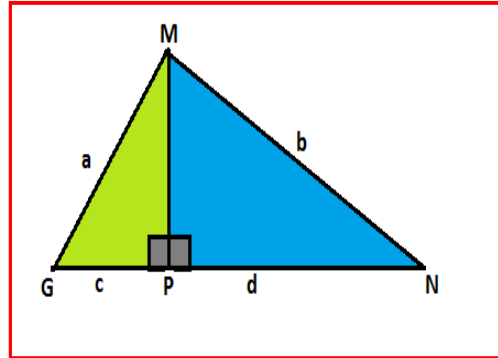
$\triangle MPN$ ಯಲ್ಲಿ, $\angle MPN = 90^\circ$,

$$\therefore MP^2 = b^2 - d^2 \text{-----(2)}$$

$$\therefore a^2 - c^2 = b^2 - d^2 \text{ [From 1 and 2]}$$

$$\therefore a^2 - b^2 = c^2 - d^2$$

$$\therefore (a + b)(a - b) = (c + d)(c - d)$$



2. In $\triangle ABC$, $\angle ABC = 90^\circ$, $BD \perp AC$. If $AB = 'c'$ units, $BC = 'a'$ units, $BD = 'p'$ units, $CA = 'b'$ units. Prove that $\frac{1}{a^2} + \frac{1}{c^2} = \frac{1}{p^2}$

$$\text{Area } \triangle ABC = \frac{1}{2} \times BC \times AB$$

$$\text{Area } \triangle ABC = \frac{ac}{2} \text{-----(1)}$$

$$\text{Area } \triangle ABC = \frac{1}{2} \times AC \times BD$$

$$\text{Area } \triangle ABC = \frac{bp}{2} \text{-----(2)}$$

$$\therefore \frac{ac}{2} = \frac{bp}{2} \text{ [From 1 and 2]}$$

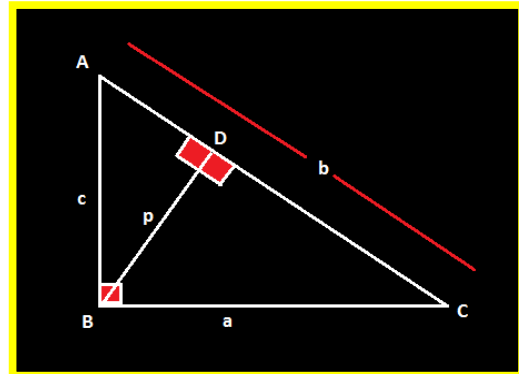
$$\Rightarrow ac = bp \Rightarrow p = \frac{ac}{b} \Rightarrow \frac{1}{p} = \frac{b}{ac} \text{-----(3)}$$

Area $\triangle ABC$, $\angle ABC = 90^\circ$

$$\therefore b^2 = a^2 + c^2$$

$$\therefore \frac{b^2}{a^2 c^2} = \frac{a^2}{a^2 c^2} + \frac{c^2}{a^2 c^2}$$

$$\therefore \frac{1}{p^2} = \frac{1}{c^2} + \frac{1}{a^2}$$



3. Derive the formula to find the height and area of an equilateral triangles.

In equilateral $\triangle ABC$, $AM \perp BC$

$\triangle AMC$ ಯಲ್ಲಿ, $\angle AMC = 90^\circ$,

$$\therefore AM^2 = AC^2 - MC^2 \text{-----(1)}$$

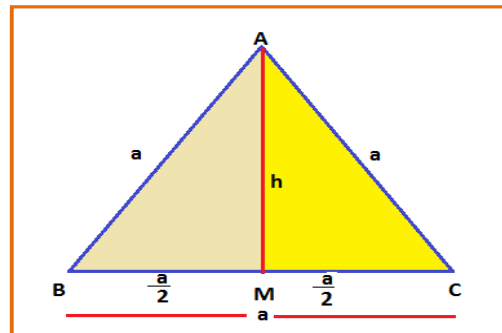
$$\Rightarrow h^2 = a^2 - \left(\frac{a}{2}\right)^2 \text{ [} \because AD \perp BC \text{]}$$

$$\Rightarrow h^2 = a^2 - \frac{a^2}{4}$$

$$\Rightarrow h^2 = \frac{4a^2 - a^2}{4}$$

$$\Rightarrow h^2 = \frac{3a^2}{4}$$

$$\Rightarrow h = \frac{\sqrt{3}a}{2}$$



$$\text{Area of } \triangle ABC = \frac{1}{2} \times BC \times AM$$

$$\Rightarrow \text{Area of } \triangle ABC = \frac{1}{2} \times a \times \frac{\sqrt{3}a}{2}$$

$$\Rightarrow \text{Area of } \triangle ABC = \frac{\sqrt{3}a^2}{4}$$

ILLUSTRATIVE EXAMPLES

Example1: Verify whether the following measures represent the sides of a right angled triangle.

(a) 6, 8, 10

Sol: Sides are : 6, 8, 10

Consider the areas of square on the sides : $6^2, 8^2, 10^2$

i.e., 36, 64, 100

Consider the sum of areas of squares on the two smaller sides : $36 + 64 = 100$

$$6^2 + 8^2 = 10^2$$

We observe that, square on the longest side of the triangle is equal to the sum of squares on the other two sides.

By converse of Pythagoras theorem, those two smaller sides must contain a right angle.

Conclusion: The sides 6, 8 and 10 form the sides of a right angled triangle with hypotenuse 10 units and 6 and 8 units as the sides containing the right angle.

Note: Without actually constructing the triangle for the given measurements of sides it is now possible to say whether the sides represent the sides of a right angled triangle using converse of Pythagoras theorem

(b) 4, 5, 6

Sides are : 4, 5, 6

Areas of squares on the sides : $4^2, 5^2, 6^2$

i.e. : 16, 25, 36

Sum of areas of squares on the two smaller sides: $16 + 25 = 41$

$$\Rightarrow 4^2 + 5^2 \neq 6^2$$

We observe that the square on the longest side of the triangle is not equal to the sum of the squares on the other two sides.

By converse of Pythagoras theorem, these two sides cannot contain a right angle. Hence, 4, 5, and 6 cannot form the sides of right angled triangle

Example2: In the quadrilateral ABCD, $\angle ABC = 90^\circ$ and $AD^2 = (AB^2 + BC^2 + CD^2)$. Prove that $\angle ACD = 90^\circ$

Sol; in $\triangle ABC$, $\angle ABC = 90^\circ$ [\because data]

$$AC^2 = AB^2 + BC^2 \text{ [}\because \text{ Pythagoras theorem]}$$

But, $AD^2 = (BD^2 + BC^2) + CD^2$ [\because data]

$$\therefore AD^2 = AC^2 + CD^2 \text{ [}\because \text{ by data } AB^2 + BC^2 = AC^2]$$

$$\therefore \angle ACD = 90^\circ \text{ [}\because \text{ Converse of Pythagoras theorem]}$$

Exercise 12.2

1. Verify whether the following measures represent the sides of a right angled triangle .

(i) 1, 2, $\sqrt{3}$ (ii) $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ (iii) $6\sqrt{3}$, 12, 6 (iv). $m^2 - n^2$, $2mn$, $m^2 + n^2$

(i) 1, 2, $\sqrt{3}$

$$1^2 = 1; 2^2 = 4; (\sqrt{3})^2 = 3$$

$$\therefore 2^2 = 1^2 + (\sqrt{3})^2$$

\therefore Not a right angled triangle.

(ii) $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$

$$(\sqrt{2})^2 = 2$$

$$(\sqrt{3})^2 = 3$$

$$(\sqrt{5})^2 = 5$$

$$\therefore (\sqrt{5})^2 = (\sqrt{3})^2 + (\sqrt{2})^2$$

\therefore This is right angled triangle

(iii) $6\sqrt{3}$, 12, 6

$$(6\sqrt{3})^2 = 36 \times 3 = 108$$

$$12^2 = 144$$

$$6^2 = 36$$

$$\therefore 12^2 = (6\sqrt{3})^2 + 6^2$$

\therefore This is right angled triangle.

(iv). $m^2 - n^2$, $2mn$, $m^2 + n^2$

$$(m^2 - n^2)^2 = (m^2)^2 + (n^2)^2 - 2m^2n^2$$

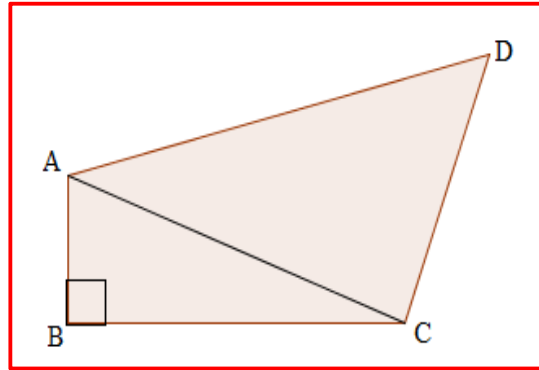
$$(m^2 - n^2)^2 = m^4 + n^4 - 2m^2n^2$$

$$(m^2 + n^2)^2 = (m^2)^2 + (n^2)^2 + 2m^2n^2$$

$$(m^2 + n^2)^2 = m^4 + n^4 + 2m^2n^2$$

$$(2mn)^2 = 4m^2n^2$$

$$\therefore (m^2 + n^2)^2 = (m^2 - n^2)^2 + (2mn)^2$$



2. In ΔABC , $a + b = 18$ units, $b + c = 25$ units and $c + a = 17$ units . what type of ΔABC ? Give reason.

$$a + b = 18$$

$$b + c = 25$$

$$c + a = 17$$

$$\Rightarrow 2a + 2b + 2c = 60$$

$$\Rightarrow 2(a + b + c) = 60$$

$$\Rightarrow (a + b + c) = 30$$

$$\therefore 18 + c = 30 \Rightarrow c = 12$$

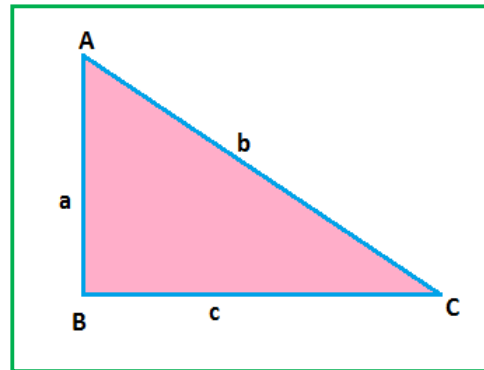
$$a + 25 = 30 \Rightarrow a = 5$$

$$b + 17 = 30 \Rightarrow b = 13$$

$\therefore \Delta ABC$ ಯಲ್ಲಿ,

$$13^2 = 5^2 + 12^2 \Rightarrow b^2 = a^2 + c^2$$

\therefore This is right angled triangle [Converse of Pythagoras theorem]



3. In ΔABC , If $CD \perp AB$, $CA = 2AD$ and $BD = 3AD$, Prove that $\angle BCA = 90^\circ$.

ΔCDA ಯಲ್ಲಿ, $\angle CDA = 90^\circ$,

$$\therefore CD^2 = CA^2 - AD^2$$

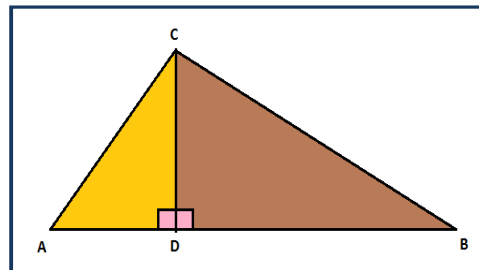
$$\Rightarrow CD^2 = (2AD)^2 - AD^2$$

$$\Rightarrow CD^2 = 4AD^2 - AD^2$$

$$\Rightarrow CD^2 = 3AD^2 \text{ -----(1)}$$

ΔCDB ಯಲ್ಲಿ, $\angle CDB = 90^\circ$,

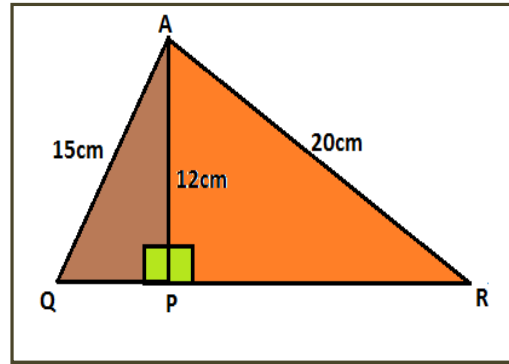
$$\therefore CD^2 = CB^2 - BD^2$$



$$\begin{aligned} \therefore CD^2 &= CB^2 - (3AD)^2 \\ \therefore CD^2 &= CB^2 - 9AD^2 \text{-----(2)} \\ \therefore 3AD^2 &= CB^2 - 9AD^2 [\because (1) \text{ ಮತ್ತು } (2)] \\ \therefore CB^2 &= 12AD^2 \text{-----(3)} \\ CA^2 &= (2AD)^2 \\ \Rightarrow CA^2 &= 4AD^2 \text{-----(4)} \\ AB^2 &= (AD + BD)^2 \\ \Rightarrow AB^2 &= (AD + 3AD)^2 \\ \Rightarrow AB^2 &= (4AD)^2 \\ \Rightarrow AB^2 &= 16AD^2 \text{----- (5)} \\ \therefore AB^2 &= CB^2 + CA^2 [\because \text{From (3), (4) and (5) }] \\ \therefore \angle BCA &= 90^\circ \end{aligned}$$

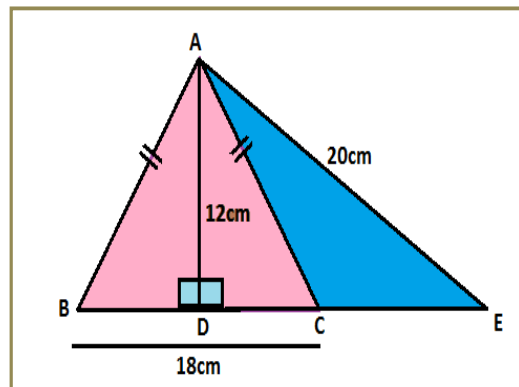
4. The shortest distance AP from a point A to QR is 12cm. 'Q' and 'R' are respectively 15cm and 20cm from 'A' and on opposite side of AP. Prove that $\angle QAR = 90^\circ$

$$\begin{aligned} \Delta APQ \text{ ನಲ್ಲಿ, } \angle APQ &= 90^\circ [\because \text{Shortest distance} = \perp] \\ \therefore QP^2 &= AQ^2 - AP^2 [\because \text{Pythagoras theorem}] \\ \therefore QP^2 &= 15^2 - 12^2 \\ \therefore QP^2 &= 225 - 144 \\ \therefore QP^2 &= 81 \text{-----(1)} \\ \Delta APR \text{ ನಲ್ಲಿ, } \angle APR &= 90^\circ [\because \text{Shortest distance} = \perp] \\ \therefore PR^2 &= AR^2 - AP^2 [\because \text{Pythagoras theorem}] \\ \therefore PR^2 &= 20^2 - 12^2 \\ \therefore PR^2 &= 400 - 144 \\ \therefore PR^2 &= 256 \text{-----(2)} \\ AQ^2 &= 15^2 = 225 \text{-----(3)} \\ AR^2 &= 20^2 = 400 \text{-----(4)} \\ QR^2 &= (QP + PR)^2 \\ QR^2 &= QP^2 + PR^2 + 2QP \cdot PR \\ \therefore QR^2 &= 81 + 256 + 2 \times 9 \times 16 \\ \therefore QR^2 &= 81 + 256 + 288 \\ \therefore QR^2 &= 625 \text{-----(5)} \\ \therefore QR^2 &= AQ^2 + AR^2 [\because \text{From (3),(4)and (5) }] \end{aligned}$$



5. In the isosceles ΔABC , $AB = AC$, $BC = 18\text{cm}$, $AD = 12\text{cm}$, BC is produced to 'E' and $AE = 20\text{cm}$. Prove that $\angle BAE = 90^\circ$.

$$\begin{aligned} \Delta ABC \text{ ಯಲ್ಲಿ, } AB &= AC, BC \perp AD \\ \therefore BD &= CD = 9\text{cm} \\ \text{In } \Delta ADC, \angle ADC &= 90^\circ \\ \therefore AC^2 &= AD^2 + CD^2 [\because \text{Pythagoras theorem}] \\ \therefore AC^2 &= 12^2 + 9^2 = 144 + 81 \\ \therefore AC^2 &= 225 \\ \therefore AB^2 &= 225 \text{-----(1)} \\ AE^2 &= 20^2 \\ AE^2 &= 400 \text{-----(2)} \\ \therefore 20^2 &= 12^2 + DE^2 [\because \text{Pythagoras theorem}] \\ \therefore 400 &= 144 + DE^2 \\ \therefore DE^2 &= 256 \\ \therefore DE &= 16\text{cm} \end{aligned}$$



$$\begin{aligned} \therefore BE &= BD + DE \\ \therefore BE &= 9 + 16 = 25\text{cm} \\ \therefore BE^2 &= 625 \text{ -----(3)} \end{aligned}$$

In $\triangle ABC$,

$$\therefore BE^2 = AB^2 + AE^2 \text{ [} \because \text{ From (1), (2) and (3)]}$$

$$\therefore \angle BAE = 90^\circ \text{ [} \because \text{ Converse of Pythagoras theorem.]}$$

6. In the quadrilateral ABCD, $\angle ADC = 90^\circ$, $AB = 9\text{cm}$, $AD = 6\text{cm}$, $BC = AD = 6\text{cm}$ and $CD = 3\text{cm}$, Prove that $\angle ACB = 90^\circ$.

$\triangle ADC$ ಯಲ್ಲಿ, $\angle ADC = 90^\circ$

$$\therefore AC^2 = AD^2 + CD^2 \text{ [} \because \text{ Pythagoras theorem.]}$$

$$\therefore AC^2 = 6^2 + 3^2$$

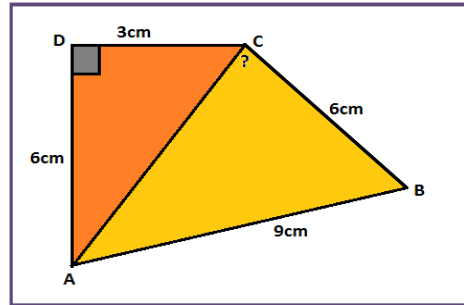
$$\therefore AC^2 = 36 + 9 = 45 \text{ -----(1)}$$

$$AB^2 = 9^2 = 81 \text{ -----(2)}$$

$$BC^2 = 6^2 = 36 \text{ -----(3)}$$

$$\therefore AB^2 = AC^2 + BC^2 \text{ [} \because \text{ From (1), (2) and (3)]}$$

$$\therefore \angle ACB = 90^\circ \text{ [} \because \text{ Converse of Pythagoras theorem]}$$



7. ABCD is a rectangle. 'P' is any point outside it such that $PA^2 + PC^2 = BA^2 + AD^2$. Prove that $\angle APC = 90^\circ$.

ABCD is a rectangle

$$\therefore AC^2 = DC^2 + AD^2$$

$$\Rightarrow AC^2 = BA^2 + AD^2 \text{ [} \because \text{ BA = DC]}$$

$$\text{But, } PA^2 + PC^2 = BA^2 + AD^2$$

$$\therefore PA^2 + PC^2 = AC^2$$

$$\therefore \angle APC = 90^\circ \text{ [} \because \text{ Converse of Pythagoras theorem]}$$

